Numerical Solution of Coupling Between Two Collinear Parallel-Plate Waveguides

Y. E. ELMOAZZEN, MEMBER, IEEE, AND LOTFOLLAH SHAFAI, MEMBER, IEEE

Abstract—The problem of coupling between two collinear parallel-plate waveguides is investigated numerically using moment methods. The exciting mode in the waveguide is assumed as the incident field, and the integral equation for the induced current is expressed in terms of the reflected, transmitted, and evanescent currents on the waveguides. The integral equation is then solved numerically by a point-matching method and the reflection and the transmission coefficients and the radiated fields are obtained. To examine the accuracy of the results, the special case of a semi-infinite exciting waveguide coupled to a finite coupled waveguide is also considered and is solved numerically by treating the singularities of the induced currents using a transformation method. For a $TE_{0,1}$ excitation of the exciting waveguide, the results of both numerical methods are compared with the analytical results obtained previously using the Wiener–Hopf technique, and are found to be in good agreement. The methods are then used to study the effect of the coupled waveguide on the radiation field.

I. INTRODUCTION

NUMERICAL techniques have recently been used to investigate certain scattering and antenna problems, for which an exact analytical or approximate solution is not readily obtainable. Two-dimensional problems involving perfectly conducting cylinders were studied by Mei and Van Bladel [1] and Andreasen [2] by solving the integral equation for the induced currents using the moment method. The scattering by the dielectric cylinders was investigated by Richmond [3], [4] where a solution is obtained by a numerical evaluation of the integral equations for the polarization currents in the dielectric. Similar scattering as well as antenna problems were also investigated by various authors and are summarized later by Harrington [5] and Mittra [6].

In general, the numerical evaluation of the integral equations is carried out by using a moment or a point-matching method to convert the integral equations to a set of linear simultaneous equations. The number of simultaneous equations for a desired degree of accuracy usually depends on the electrical length of the cross-sectional contour for the conducting cylinders and on the cross-sectional area of the dielectric cylinders. However, for problems involving discontinuities in the cross-sectional contour a larger number of matching points may become necessary if the induced current had a singular behavior near the discontinuities. For an accurate solution of these types of problems a treatment of the singularities which can effectively reduce the required number of matching points for a desired degree of accuracy may be necessary. To remove these singularities Abdelmessih and Sinclair [7] have used Meixner's edge conditions to describe the behavior of the currents near a discontinuity. An alternative method based on the coordinate transformation has also been used by Shafai [8] where the singular behavior of the current is related directly to the geometry of the scatterer. The convergence of the solution may also be improved through the use of *a priori* knowledge of the solution [9]. Here the unknown solution may be assumed as the sum of a known approximate solution and an unknown perturbation function, which is then found by an application of the moment method.

In the previously mentioned papers the moment method has been applied to solve scattering problems of obstacles with a finite cross-sectional area. The extension of the method to two-dimensional scattering by obstacles of semi-infinite but arbitrary cross sections has recently been investigated by Morita [10], [11]. The induced current is assumed to be the sum of known geometrical optics and unknown diffraction current, and the unknown current is found numerically. This method was later used by Wu and Chow [12] to investigate the problem of discontinuities inside semi-infinite waveguides. They have expressed the induced currents in terms of propagating and evanescent modes and have found evanescent currents numerically, by using their localized nature, in order to apply the moment method for a finite section near the discontinuity.

In this paper the approach of Wu and Chow is used to formulate the coupling between two semi-infinite waveguides. The induced current is expressed in terms of incident, reflected, transmitted, and evanescent currents which are found by an application of the point-matching method. The method is then applied to the case where the coupled waveguide is of finite length. To examine the accuracy of the solution the transformation method is

Manuscript received July 1, 1974; revised May 20, 1975. This work was supported in part by the National Research Council of Canada (Grant A-7702), and in part by the Department of Electrical Engineering, University of Manitoba. This paper is part of a dissertation submitted to the Department of Electrical Engineering, Faculty of Graduate Studies, University of Manitoba, Winnipeg, Man., Canada, in partial fulfillment of the requirements for the Ph.D. degree.

Y. E. Elmoazzen was with the Department of Electrical Engineering, University of Manitoba, Winnipeg, Man., Canada R3T, 2N2. He is now with The Department of Telecommunication, Northern Alberta Institute of Technology, Edmonton, Alta., Canada.

L. Shafai is with the Department of Electrical Engineering, University of Manitoba, Winnipeg, Man., Canada R3T 2N2.

also used to investigate the latter case of the finite-coupled waveguide and the computed results are compared with those of the moment method and the analytic results of the Wiener–Hopf technique [13]. The methods are then used to study the effect of each induced current and the coupled waveguide on the radiation field. It is shown that the coupled waveguide has a significant effect on the radiation field and its pattern can be modified by changing the separation and the length of the coupled waveguide.

II. FORMULATION OF THE PROBLEM

A. The Moment Method

A geometry of the problem is shown in Fig. 1. Assuming a $TE_{0,1}$ mode to propagate in the exciting waveguide, the current densities, due to the incident wave on the lower and the upper walls, are equal and are given by (the time factor $\exp(i\omega t)$ is understood)

$$J_z{}^i = \hat{z}\,\frac{\sin\theta_0}{\eta}\exp(ikx\cos\theta_0) \tag{1}$$

where $\theta_0 = \sin^{-1}(\lambda/2d)$, d is the waveguide width, and η is the intrinsic impedance of the medium inside the waveguide. Due to the discontinuity at $x = 0$, part of the incident field reflects back into the exciting waveguide and contributes to the reflected field. The remaining part undergoes a diffraction at the open end of the exciting waveguide and contributes partly to the radiation field and partly undergoes further diffraction at the open end of the coupled waveguide. The multiple diffractions between two waveguides give rise to the reflected and the transmitted fields in the waveguides, which induce current densities in the form

$$J_z{}^r = \hat{z}R\,\frac{\sin\theta_0}{\eta}\exp(-ikx\cos\theta_0), \qquad x > 0, \qquad y = 0,d \tag{2}$$

$$J_z{}^t = \hat{z}T\,\frac{\sin\theta_0}{\eta}\exp(ikx\cos\theta_0), \qquad x < -L, \qquad y = 0,d \tag{3}$$

where R and T are the reflection and the transmission coefficients, respectively. The evanescent modes also contribute to the induced currents on the waveguides, but their contribution is significant only near the open end of each waveguide. Representing these evanescent currents by $J^e = \hat{z}J_z{}^e$, the boundary condition $E_z = 0$ on the walls gives the following integral equation for the induced currents [12]:

$$0 = \frac{\eta}{4}\int_W [J^e(r') + J^i(r') + J^r(r') + J^t(r')]H_0{}^{(2)} \cdot (k\,|r - r'|)\,d(kr') \tag{4}$$

where r and r' are the coordinates of the field and source points on the walls of the waveguides. The integration path W is along the walls, which support the respective current distribution. Since this integral equation can not be solved exactly, a moment method will be used to reduce the integral equation to a set of linear simultaneous equations. However, the integrals involving J^i, J^r, and J^t can be evaluated exactly which gives the excitation and the coefficients R and T. For the evanescent currents J^e, using unit pulses for the base function, the waveguide walls may be divided into N segments with a current $J_n{}^e$ at the center of each segment. Since the evanescent currents are significant only near the open ends, the N segment may be chosen between $x = 0$ and $x = x_a$ and between $x = -L$ and $x = x_b$. Equation (4) then contains $N + 2$ unknowns which, with $N + 2$ test points, can be written in a matrix form as

$$[l_{m,n}][f_m] = [g_m] \tag{5}$$

where

$$f_m = \begin{cases} (\eta/4)J_m{}^e, & m = 1,2,\cdots,N \\ R, & m = N+1 \\ T, & m = N+2 \end{cases} \tag{6}$$

$$g_m = -\frac{\sin\theta_0}{4}[I_+(0,\infty,0) + I_+(0,\infty,kd)] \tag{7}$$

and the elements of the matrix $l_{m,n}$ are given as follows.

1) For $m = 1,2,\cdots,N + 2$ and $n = 1,2,\cdots,N$

$$l_{m,n} = \begin{cases} [H_0{}^{(2)}(k\,|x_m - x_n'|) + H_0{}^{(2)}(k[(x_m - x_n')^2 + d^2]^{1/2})]\Delta(kx_n'), & m \neq n \\ [1 - i(2/\pi)(\ln[\Delta(kx_n')/4] + \gamma - 1) + H_0{}^{(2)}(kd)]\Delta(kx_n'), & m = n \end{cases} \tag{8}$$

where $\gamma = 0.5772\cdots$.

2) For $m = 1,2,\cdots, N + 2$ and $n = N + 1$

$$l_{m,N+1} = \frac{\sin\theta_0}{4}[I_-(0,\infty,0) + I_-(0,\infty,kd)]. \tag{9}$$

3) For $m = 1,2,\cdots, N + 2$ and $n = N + 2$

$$l_{m,N+2} = \frac{\sin\theta_0}{4}[I_+(-\infty,-L,0) + I_+(-\infty,-L,kd)]. \tag{10}$$

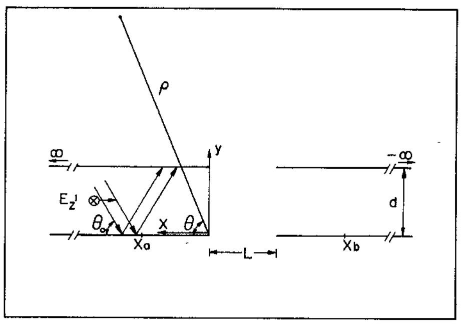

Fig. 1. Geometry of the problem.

The integral I in these equations is defined by

$$I_{\pm}(A,B,Y) = \int_A^B \exp(\pm ikx' \cos\theta_0) H_0^{(2)} \{[(kx' - kx_m)^2 + (ky)^2]^{1/2}\} \, d(kx'). \quad (11)$$

An evaluation of the integrals in these equations gives the required matrix elements of (5) and involves lengthy manipulations. The details are omitted here, but can be found in [16]. Now a solution of (5) gives the evanescent current J^e, the reflection coefficient R, and the transmission coefficient T. This completes the solution of the coupling between two waveguides. However, it does not provide the radiated field of the exciting waveguide. This radiation field can be obtained by introducing the results of (5) into the integral equation (4), for a point away from the waveguide walls, and by evaluating the resulting integrals. For points at large distances from the open ends of the waveguides, it may be shown that the radiation field is given by

$$E_z(r) \underset{\rho\to\infty}{=} \left(\frac{2}{\pi k\rho}\right)^{1/2} \exp[-i(k\rho - \pi/4)] F(\theta) \quad (12)$$

where

$$F(\theta) = [1 + \exp(ikd)\sin\theta] \left\{ \frac{i}{4} \frac{\sin\theta_0}{\cos\theta + \cos\theta_0} \cdot \{1 - T\exp[-ikL(\cos\theta - \cos\theta_0)]\} + \frac{iR}{4} \frac{\sin\theta_0}{\cos\theta - \cos\theta_0} + \sum_{n=1}^{N} \frac{\eta}{4} J_n^e \cdot \exp(ikx_n\cos\theta)\Delta(kx_n) \right\}. \quad (13)$$

This equation gives the radiation field due to the coupling between two semi-infinite waveguides as a function of the reflection and the transmission coefficients and the evanescent currents. In the absence of the coupled waveguide the transmission coefficient is zero and the reflection coefficient is that of a semi-infinite waveguide. The system of the simultaneous equations is then $J_1^e, \cdots, J_N^e$ and R for $N + 1$ unknowns.

Similarly, if the coupled waveguide is not a semi-infinite type but has a finite length L_2 the induced currents on its walls may be represented by a single unknown current $J = J^e + J^t$. The system of simultaneous equations is again for $N + 1$ unknowns, in which R is the reflection coefficient in the presence of a finite coupled waveguide.

B. Transformation Method

The moment method described in the previous section provides a general formulation for the numerical solution of the coupling between two waveguides. However, the accuracy of the solution will depend on the behavior of the evanescent currents and on the location of testing points for the reflection and the transmission coefficients. Therefore, to examine the accuracy of the solution, the transformation method, which can give more accurate solution, is also used, provided the computation time is not excessive. Thus, to simplify the problem, only the case of a coupled waveguide with a finite length will be considered. Furthermore, if the separation distance kL between the waveguides is large enough, the interaction between the waveguides may be neglected and the total radiation field may be assumed as the sum of the radiation field from the open end of the exciting waveguide and the field scattered by the coupled waveguide. The problem then reduces to a two-dimensional scattering by two parallel conducting strips, separated by a distance d.

For a $TE_{0,1}$ mode propagating in the exciting waveguide, the radiation field is given by [13] [with a time factor $\exp(-i\omega t)$]

$$E_z \underset{\rho\to\infty}{=} \frac{i\pi}{2a(2\pi k\rho)^{1/2}} \exp\left\{ i(k\rho - ka\,|\sin\theta| - \tfrac{1}{4}\pi) \cdot \frac{|\sin\theta|\, G_+(k\cos\theta_0) G_+(k\cos\theta)}{\cos\theta + \cos\theta_0} \right\} \quad (14)$$

where $a = d/2$ is the half-width of the waveguides and $G_+(\alpha)$ is given by [13]

$$G_+(\alpha) = \left(\frac{\cos ka}{k+\alpha}\right)^{1/2} \exp\left\{ i\frac{\pi}{4} + i\left(\frac{\alpha a}{\pi}\right) \cdot \left[1 - c + \ln\left(\frac{\pi}{2ka}\right) + i\frac{\pi}{2}\right] + i\frac{\gamma a}{\pi} \ln\left(\frac{\alpha - \gamma}{\pi}\right) \right\} \prod_{n=1,3,5\cdots}^{\infty} \left(1 + \frac{\alpha}{i\gamma_n}\right) \exp\left(i\frac{2\alpha a}{n\pi}\right) \quad (15)$$

where

$$c = 0.57721, \qquad \gamma_n = [(n\pi/2a)^2 - k^2]^{1/2},$$

$$k\cos\theta_0 = j\gamma_1 \quad \text{and} \quad \gamma = [\alpha^2 - k^2]^{1/2}.$$

This radiation field induces a current J_z on the walls of the coupled waveguide, which together with the above radiation field gives a total radiation field in the form

$$E_z^{\text{total}}(r) = E_z^{\text{inc}}(r) + \tfrac{1}{4}\eta \int_W H_0^{(1)}(k\,|r - r'|) J_z(r')\, d(kr') \quad (16)$$

where E_z^{inc} is given by (14). The integration path W is along the walls of the coupled waveguide and r and r' are the coordinates of the field and source points, respectively. If r is on the walls of the coupled waveguide, the boundary condition $E_z^{\text{total}} = 0$ reduces (16) to an integral equation for the current J_z which can be solved by a point-matching method. However, for the previously mentioned polarization of the incident field, the induced current J_z is singular near the edges of the coupled waveguide. Thus, for an accurate evaluation of J_z a transformation may be used to introduce an auxiliary function inside the integral which, together with J_z, provides a regular function. Since the walls of the coupled waveguide are basically strips of finite size, the conformal transformation of the strip on to a circular cylinder is the most convenient transformation. Such a transformation introduces a metric coefficient h inside the integral, the reciprocal of which has a singular behavior identical to that of the induced current. The detail of such a transformation was previously discussed by Shafai [8], [15] and its application to the current problem is shown in [16]. For the current problem the induced currents on the walls are equal and the transformation modifies the integral equation to the form

$$E_z^{\text{inc}}(r) = \tfrac{1}{4}\eta \int_0^{2\pi} [H_0^{(1)}(k \mid r_0 - r' \mid) + H_0^{(1)}(k \mid r_0 - r'' \mid)] I_z(\phi')\, d\phi' \quad (17)$$

where r' and r'' are the coordinates of integration points on the upper and the lower walls, respectively, and ϕ' is a variable in the transformed domain. The function I_z is the new unknown function given by $h_\phi J_z$ and is regular. A numerical solution of (17) can be obtained by any convenient method. However, since I_z is regular and generally well behaved, it may be expressed by a Fourier series of the ϕ' coordinate with unknown coefficients. For $TE_{0,1}$ excitation the illumination is symmetric with respect to each strip and I_z can be assumed to be

$$I_z(\phi') = \sum_{n=0}^{\infty} a_n \cos n\phi' \quad (18)$$

and the integral equation (17) becomes

$$E_z^{\text{inc}}(r_0) = \tfrac{1}{2}\eta \sum_{n=0}^{\infty} a_n \int_0^{\pi} [H_0^{(1)}(k \mid r_0 - r' \mid) + H_0^{(1)}(k \mid r_0 - r'' \mid)] \cos n\phi'\, d\phi'. \quad (19)$$

Choosing N terms from the series, N matching points on any wall of the coupled waveguide reduces the foregoing equation to a set of N simultaneous equations, the solution of which gives N unknown coefficients a_n. Practically, the preceding Fourier expansion has some advantages over a moment solution. Each intermediate integral can be evaluated with any desired degree of accuracy (an accurate solution for I_z can be found by choosing adequate terms within the stable region of the numerical solution) and provides an analytical function for I_z.

Once the coefficients a_n are obtained, the far scattered field can be found from

$$E_z^{\text{sc}}(r) \underset{\rho\to\infty}{=} -\tfrac{1}{2}\eta \sum_{n=0}^{N=1} a_n \int_0^{\pi} \cos n\phi' \cdot (\exp\{ik\rho'[\cos(\theta+\theta') + \cos(\theta-\theta')]\})\, d\phi' \cdot \left(\frac{2}{\pi k\rho}\right)^{1/2} \exp[+i(k\rho - \pi/4)] \quad (20)$$

where

$$\rho' = [x'^2(\phi') + y'^2(\phi')]^{1/2}$$
$$\theta' = \tan^{-1}[y'(\phi')/x'(\phi')]. \quad (21)$$

The total radiated field is then obtained by adding the incident and the scattered fields. The results of the numerical computations are presented in the next section.

III. RESULTS AND DISCUSSION

For a $TE_{0,1}$ mode propagating in the exciting waveguide, and using the moment method, some numerical results are obtained and are discussed in this section. To examine the contribution of each induced current to the radiation field, the radiation from a single semi-infinite parallel-plate waveguide is considered and the computed results are shown in Fig. 2(a) and (b). The total radiation fields are in good agreement with the results of analytical solution using the Wiener–Hopf solution [14]. The contribution of the reflected and evanescent currents to the radiation fields are also shown and have similar behavior with the main radiation in the forward direction.

Fig. 3 shows the radiation patterns calculated by the moment method for a finite coupled waveguide of length $kL_2 = 15$ and $d/\lambda = 0.60$. For the small separation distance $kL = 0.1$ the resulting pattern is almost the same as that of a semi-infinite waveguide. This is due to the fact that the main radiation in this case is from the open end of the coupled waveguide. The reflection coefficients for the previous cases are shown in Table I. Again the magnitude of the reflection coefficient for $kL = 0.1$ is the same as that of a single semi-infinite waveguide. The phase angle, however, is different, since it represents ap-

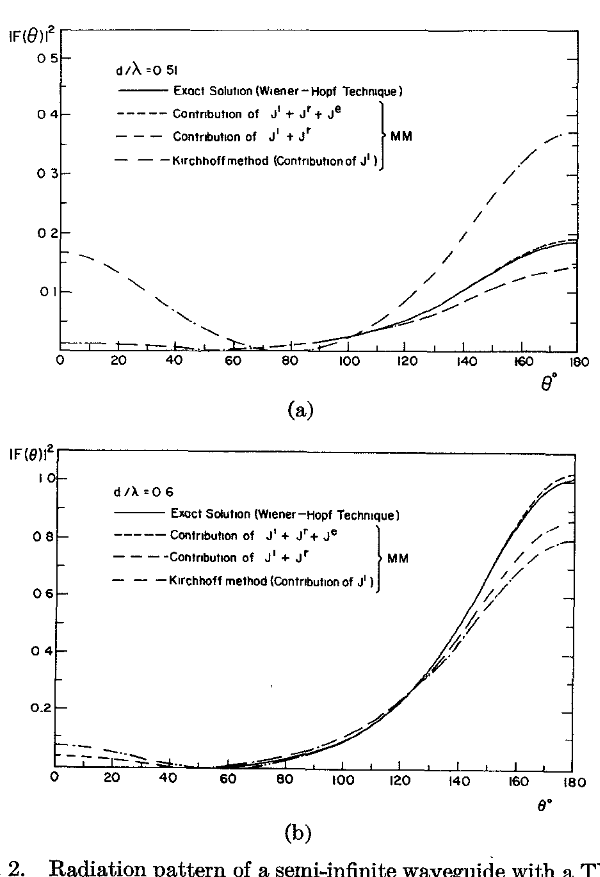

Fig. 2. Radiation pattern of a semi-infinite waveguide with a $TE_{0,1}$ excitation.

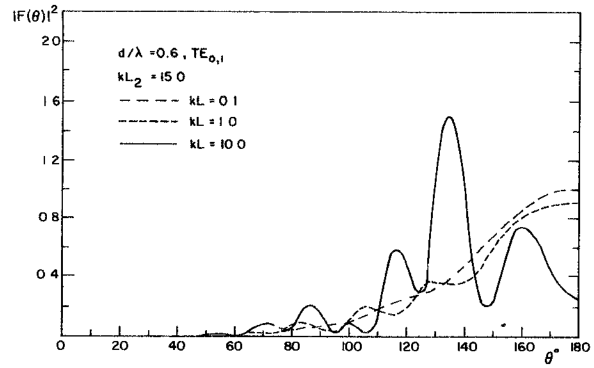

Fig. 3. Radiation pattern of two collinear waveguides by moment method, finite coupled waveguide, and $TE_{0,1}$ excitation.

TABLE I

Reflection Coefficient by MM for Two Collinear Parallel-Plate Waveguides of Finite Length $d/\lambda = 0.6$ and $kL_2 = 15$

KL	Magnitude	phase in degrees
10	0.233	-103.6
1.0	0.325	148.1
0.1	0.198	85.1

proximately the reflection from the far end of the coupled waveguide. A comparison between the reflection coefficients computed by the moment method and those obtained using the Wiener–Hopf technique [16] is shown in Table II. The agreement between the results of two methods is reasonable and improves as kL increases. This may be due to the fact that the Wiener–Hopf solution was originally obtained by assuming a large separation distance kL. Consequently, its accuracy improves as kL increases.

To examine the accuracy of the moment solution, the previous case of a finite waveguide with $kL_2 = 15$ is also solved with the transformation method. Fig. 4 shows the radiation patterns for $kL = 50$ and $d/\lambda = 0.6$ obtained by both numerical methods and the Wiener–Hopf technique. The agreement between the results of the moment and the transformation methods is fairly good. The small difference between the results should be due to the interaction between two waveguides which was neglected in the case of the transformation method and computational errors in the moment solution. The Wiener–Hopf solution is slightly different in the forward direction, which again might be due to some edge interactions neglected in the Wiener–Hopf derivation [16]. Additional results for the radiation patterns using the transformation method are also obtained and are shown in Fig. 5. The results for different values of kL_2 oscillate with θ, the azimuthal angle, around the pattern of a single semi-infinite waveguide. The amplitude of the oscillation increases with kL_2 due to the partial resonance of the coupled waveguide.

Finally, for the case of two parallel-plate semi-infinite waveguides the radiation patterns for $d/\lambda = 0.6$ and $kL = 0.1$, 10, and 50 are also obtained and are shown in Fig. 6. For $kL = 10$ and 50 the results are compared with those of the Wiener–Hopf technique and show fairly good agreement except in the forward direction. These discrepancies should be due to the computational errors in the moment method which resulted in a finite radiation

TABLE II

Reflection Coefficient for the Case of a Coupled Waveguide of Length L_2 $d/\lambda = 0.6$ and $kL_2 = 15$

KL	Using Wiener-Hopf Technique		Using MM	
	Magnitude	phase in degrees	Magnitude	phase in degrees
10	0.200	-113	0.233	-103.6
20	0.168	-123	0.151	-113
50	0.199	-133	0.194	-142

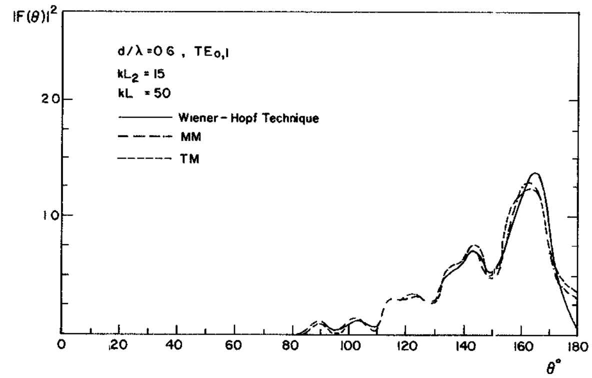

Fig. 4. Comparison of radiation patterns by different methods.

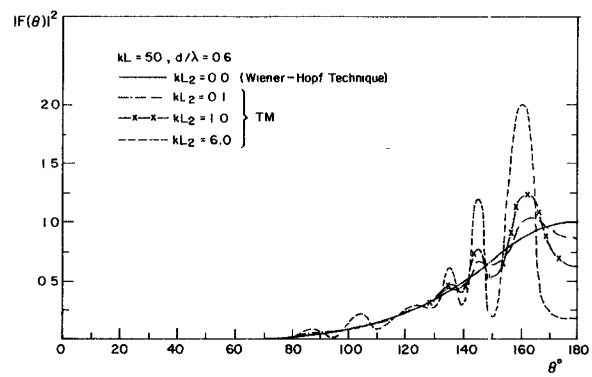

Fig. 5. Radiation patterns for different lengths of coupled waveguides, $TE_{0,1}$ excitation.

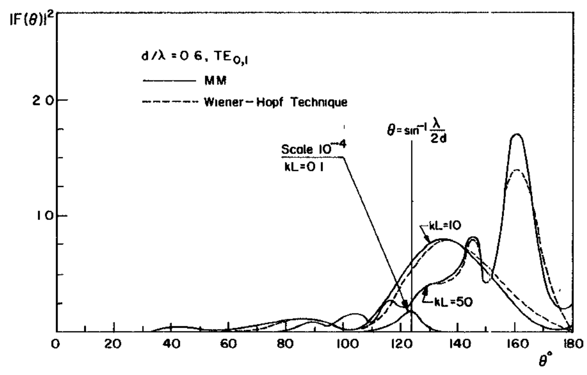

Fig. 6. Radiation pattern of two collinear semi-infinite waveguides.

TABLE III

COMPARISON BETWEEN WIENER–HOPF TECHNIQUE AND MM FOR REFLECTION AND TRANSMISSION COEFFICIENTS OF THE $TE_{0,1}$ MODE

KL	Reflection Coefficient		Transmission Coefficient	
	Wiener-Hopf Tech.	MM	Wiener-Hopf Tech.	MM
0.1	-	0.0015/-103	-	0.999/ .5
1.0	-	0.116/131	-	0.991/7.5
10.0	0.236/-111	0.285/-113	0.429/-148	0.501/-159
50.0	0.199/-135	0.192/-135	0.195/147	0.261/140

field at $\theta = 180°$. (It was found that the results of the moment method were somewhat sensitive to the locations of the testing points for the reflection and the transmission coefficients.) The computed reflection and the transmission coefficients using the moment and the Wiener–Hopf techniques are shown in Table III. The agreement again improves as kL increases.

In conclusion, the moment method was used to study the coupling between two collinear parallel-plate waveguides. An integral equation for the induced currents on the walls was obtained and was solved to give the reflection and the transmission coefficients and the evanescent currents. Formulation was obtained for the special case of two collinear waveguides of equal size, but the results can be readily extended to the case of two waveguides of different sizes and orientation. To examine the accuracy of the solution, the computed results were compared with those of the Wiener–Hopf technique and good agreement was obtained. For large separation of the waveguides, with a finite coupled waveguide, a transformation method was also used and results in good agreement with those of the moment method were obtained.

In comparison, the moment method is more general and can yield reasonable solutions for waveguides of different sizes and orientations. However, it is somewhat sensitive to the location of the testing points in numerical solution of the integral equation. The transformation method is more accurate, but was restricted here to large separations and finite coupled waveguides. The Wiener–Hopf technique discussed previously gives an analytical solution, but its application to waveguides of different size is too complex.

REFERENCES

[1] K. K. Mei and J. G. Van Bladel, "Scattering by perfectly-conducting rectangular cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-11, pp. 185–192, Jan. 1963.

[2] M. G. Andreasen, "Scattering from parallel metallic cylinders with arbitrary cross sections," *IEEE Trans. Antennas Propagat.*, vol. AP-12, pp. 746–754, Nov. 1964.

[3] J. H. Richmond, "Scattering by a dielectric cylinder of arbitrary cross section shape," *IEEE Trans. Antennas Propagat.*, vol. AP-13, pp. 334–341, May 1965.

[4] ——, "TE-wave scattering by a dielectric cylinder of arbitrary cross-section shape," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 460–464, July 1966.

[5] R. F. Harrington, *Field Computation by Moment Methods.* New York: Macmillan, 1968.

[6] R. Mittra, Ed., *Computer Techniques for Electromagnetics.* New York: Pergamon, 1973.

[7] S. T. M. Abdelmessih and G. Sinclair, "Treatment of singularities in scattering from conducting polygonal cylinders," *Can. J. Phys.*, vol. 45, pp. 1305–1318, 1967.

[8] L. Shafai, "An improved integral equation for the numerical solution of diffraction problems," *Can. J. Phys.*, vol. 48, p. 953, 1970.

[9] M. D. Tew and L. L. Tsai, "A method toward improved convergence of moment-method solutions," *Proc. IEEE* (Lett.), vol. 60, pp. 1436–1437, Nov. 1972.

[10] N. Morita, "Diffraction by arbitrary cross-sectional semi-infinite conductor," *IEEE Trans. Antennas Propagat.*, vol. AP-19, pp. 358–364, May 1971.

[11] ——, "Comments on 'Diffraction by arbitrary cross-sectional semi-infinite conductors,'" *IEEE Trans. Antennas Propagat.*, vol. AP-20, pp. 517–519, July 1972.

[12] S. C. Wu and Y. L. Chow, "An application of the moment method to waveguide scattering problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 744–749, Nov. 1972.

[13] Y. E. Elmoazzen and L. Shafai, "Mutual coupling between parallel-plate waveguides," *IEEE Trans. Microwave Theory Tech.* (*1973 Symp. Issue*), vol. MTT-21, pp. 825–833, Dec. 1973. ——, "Correction to 'Mutual coupling between parallel-plate waveguides,'" *ibid.*, vol. MTT-22, p. 816, Aug. 1974.

[14] L. A. Weinstein, *The Theory of Diffraction and the Factorization Method.* Boulder, Colo.: Golem, 1969.

[15] L. Shafai, "Current induced on a conducting strip," *Can. J. Phys.*, vol. 49, pp. 495–498, 1971.

[16] Y. E. Elmoazzen, "Coupling between two collinear waveguides," Ph.D. dissertation, Univ. Manitoba, Winnipeg, Man., Canada, 1974.